Δ-scaling and heat capacity in relativistic ion collisions

Y. G. Ma†[†], G. L. Ma†‡, X. Z. Cai†, J. G. Chen†, J. H. Chen†, D. Q. Fang†, W. Guo†, Z. J. He†, H. Z. Huang§, J. L. Long†, C. W. Ma†, B. H. Sa‖, W. Q. Shen†, Q. M. Su†, K. Wang†, Y. B. Wei†, T. Z. Yan†, C. Zhong†, J. X. Zuo†

† Shanghai Institute of Applied Physics, Chinese Academy of Sciences, China
‡ Graduate School of the Chinese Academy of Sciences, China
§Dept of Physics and Astronomy, University of California at Los Angeles, USA
‖ China Institute of Atomic Energy, P.O.Box 918, Beijing 102413, China

Abstract. The Δ-scaling method has been applied to the total multiplicity distribution of the relativistic ion collisions of p+p, C+C and Pb+Pb which were simulated by a Monte Carlo package, LUCIAE 3.0. It is found that the Δ-scaling parameter decreases with the increasing of the system size. Moreover, the heat capacities of different mesons and baryons have been extracted from the event-by-event temperature fluctuation in the region of low transverse mass and they show the dropping trend with the increasing of impact parameter.

Studying fluctuation and correlation is an important issue to investigate matter properties formed in relativistic nucleus-nucleus collisions. Some observables have been suggested to quantify the fluctuation, such as the balance function, net charge fluctuation, multiplicity fluctuation, transverse momentum fluctuation, particle ratio fluctuation and Φ_{PT} etc [1]. In this article we will discuss some novel methods. We will apply a universal fluctuation method (Δ-scaling) which was recently proposed to explore the order-disorder phase transition in the low-intermediate energy heavy ion collisions [2, 3] to discuss multiplicity fluctuation in relativistic ion collisions. In addition, the heat capacity of hadrons are extracted from the event-by-event temperature fluctuation.

Δ-scaling is observed when two or more probability distributions $P[m]$ of the stochastic observable m collapse onto a single scaling curve Φ(z) if a new scaling observable is defined by $z = \frac{(m-m^*)}{\langle m\rangle^\Delta}$. This curve is [2]:

$$\langle m\rangle^\Delta P[m] = \Phi(z) \equiv \Phi[\frac{m-m^*}{\langle m\rangle^\Delta}] \tag{1}$$

where Δ is a scaling parameter, m^* is the most probable value of m, and $\langle m\rangle$ is the mean of m. If we assume that $P[m]$ is a Gaussian distribution, we have $P[m] = \frac{1}{\sigma\sqrt{2\pi}}exp[-\frac{1}{2}(\frac{m-\mu}{\sigma})^2]$, where $\mu = \langle m\rangle = m^*$, σ is the width of the Gaussian

† To whom correspondence should be addressed: ygma@sinap.ac.cn

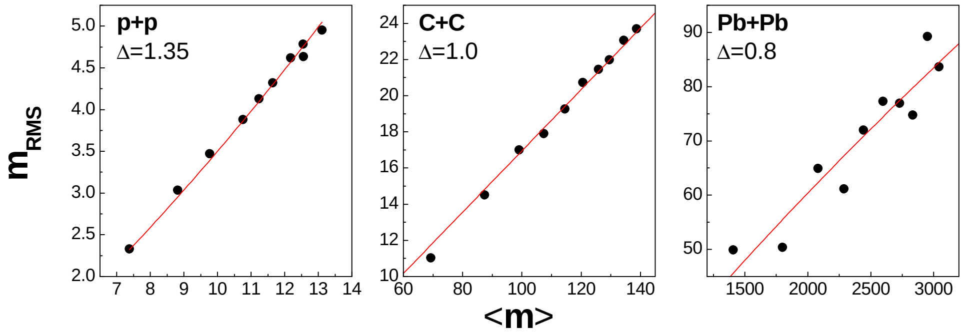

Figure 1. The determination of the Δ-scaling parameter using the power-law fits to the relationship m_{RMS} vs $\langle m\rangle$. From the left to right, it corresponds to p + p with $\Delta = 1.35$, C + C with $\Delta = 1.00$ and Pb+Pb with $\Delta = 0.80$. The circles are the calculated values and the lines represent the power-law fits.

distribution, both depending on incident energy. If this Gaussian distribution $P[m]$ obeys Δ-scaling law, we have

$$\sigma \propto \mu^{\Delta}. \tag{2}$$

In this work, p+p, C+C and Pb+Pb in SPS energies (20-200 AGeV) have been investigated with the help of LUCIAE3.0 model [4]. The head-on (b=0fm) collisions are simulated in this work. The LUCIAE is an extension Monte Carlo model of the FRITIOF [5], in which a nucleus - nucleus collision is described as the independent sum of nucleon-nucleon collisions. For more details of LUCIAE, please see Ref. [4].

In order to better investigate the Δ-scaling, we can use the power-law fit to the relationship between the RMS width of the multiplicity distribution (m_{RMS}) and the mean value of the multiplicity distribution ($\langle m\rangle$) according to Eq.(1). If there exists a good fit: $m_{RMS} = c_0\langle m\rangle^{\Delta}$ where c_0 is a fit parameter, indicating that a good Δ-scaling law is satisfied with a scaling parameter Δ. In this way, we found that the best fit value of Δ is 1.35 for p+p, 1.00 for C+C and 0.80 for Pb + Pb, respectively, which is shown in Figure 1. In the figure, each point represents an energy point from 20 AGeV to 200 AGeV with the interval of 20 AGeV. When $\Delta = 1$, the RMS width (fluctuation) is proportional to its mean value. This is the case of C+C system. For light system p+p, the rising of the RMS width is faster than its mean value with the increasing of beam energy. In contrary, for heavier system, the growing of the RMS width is slower than the mean value. In all three cases, Δ-value is larger than 1/2, which is the case of the independent particle emission (Poisson distribution). The decreasing behaviour of Δ-value with the system size reflects how the multiplicity fluctuation grows with its mean value. Its physics origin may stem from the stronger rescattering effect in larger system which induce some correlations in multiparticle production process and in consequence the fluctuation becomes weaken.

Using these values of Δ, the multiplicity distributions can be compressed onto a unique curve for a given collision system from 20 AGeV to 200 AGeV. Figure 2 shows the Δ-scaling curves for p+p, C+C and Pb+Pb systems with Δ value of 1.35, 1.00 and

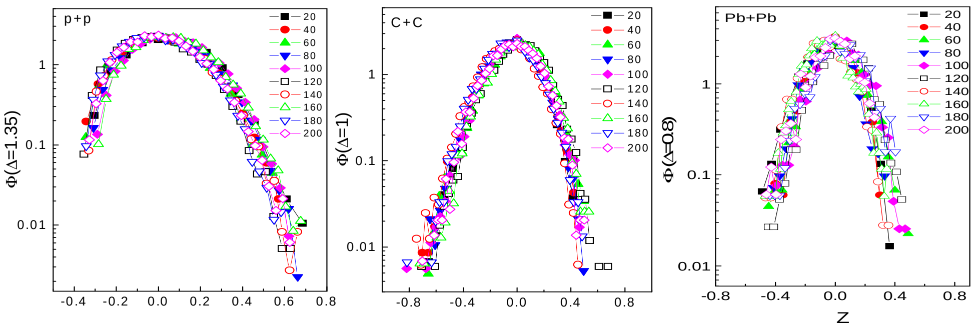

Figure 2. Δ-scaling for p + p with $\Delta = 1.35$, for C + C with $\Delta = 1.00$ and for Pb+Pb with $\Delta = 0.80$ for the different beam energies.

0.80, respectively.

In relativistic collisions, the transverse mass (m_T) distribution of the particles in lower m_T region can be approximately described as $\frac{1}{m_T}\frac{dN}{dm_T} = Ae^{-\frac{m_T}{T}}$ where A is a normalized coefficient which is related to the volume/multiplicity term and T is an apparent temperature. Thanks to the large enough particle multiplicity, it is possible to extract A and T for particles on event-by-event basis. In this way, the event-by-event temperature distribution can be constructed. Usually, such kind of temperature distribution ($P(T)$) can be described by [6, 7]

$$P(T) \sim exp[-C_v(\frac{\Delta T}{T})^2], \tag{3}$$

where ΔT is the deviation ($T - \langle T \rangle$) of temperature from the mean value ($\langle T \rangle$) and C_v can be explained as the heat capacity of a certain hadron. Since the heat capacity is an extensive observable, we define a normalized heat capacity C_v/N, i.e. the heat capacity per hadron multiplicity (N) which corresponds to the energy that is required to go up one unit temperature for producing one particle.

The heat capacity of different particles (in the range of $m_T - m_0 < 0.5$ GeV) has been extracted by Eq.(3). The left panel of Figure 3 shows an example of event-by-event temperature distribution of π^+ for the head-on Pb+Pb collisions at 160 AGeV. From this kind of distribution, we apply Eq.(3) to extract the heat capacity C_v. The curve of the figure depicts this fit. In this way, we get the heat capacity for various hadrons in different energies. The middle panel of Figure 3 gives the dependence of heat capacity of various particles (π^+, k^+, p and Λ) on incident energy in head-on Pb+Pb collision. From this figure, the normalized heat capacity of various particles decreases with the increasing of incident energy and tends to a saturation, indicating that less energy is needed to rise the same temperature in higher energies. In the viewpoint of fluctuation, this means that the increasing m_T fluctuation at higher energies. The normalized heat capacity of the particles increases with mass of particle, which reflects that more energy is needed to rise the same temperature for heavier particles. In addition, the impact parameter dependence of the heat capacity is also investigated for π^+. The right panel of Figure 3 depicts the dependence of C_v/N for π^+ on the impact parameter for Pb + Pb. The decreasing trend of C_v/N indicates that the temperature fluctuation becomes

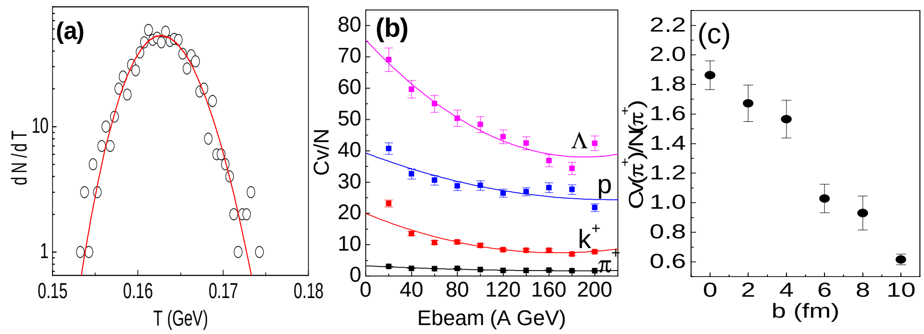

Figure 3. (a) The event-by-event temperature distribution of π^+ in the head-on Pb+Pb collision at 160AGeV. The curve is a fit with Eq.(3); (b) The normalized heat capacity of different mesons and baryons as a function of incident energy for the head-on Pb + Pb collisions. The lines represent the second polynomial fits to guide the eyes; (c) The dependence of the normalized heat capacity of π^+ on impact parameter for 160 AGeV Pb + Pb collisions. The error bars are statistical.

larger in peripheral collisions, which is similar to the P_T fluctuation or balance function as a function of centrality in STAR data [8].

In summary, we have demonstrated that there exists the Δ-scaling for the total multiplicity of charged particles for the simulated head-on collisions of p + p, C + C and Pb + Pb from E_{lab} = 20 to 200 AGeV in LUCIAE model. That means that the multiplicity distributions obey a certain kind of universal laws, regardless of beam energies and collision systems. It is found that the scaling value of the Δ decreases with the system size, which reflects that the growth of the fluctuation with the multiplicity is faster in light system. The event-by-event thermal fluctuation in lower m_T region is constructed and the normalized heat capacities of the different mesons and baryons are extracted. It is found that the heat capacity per hadron decreases with the increasing incident energy and impact parameter, while it increases with mass of particle. Considering that the LUCIAE model is a string-hadronic model, the partonic effect which becomes more important in RHIC energies is absent in the model. This effect should be investigated in near future. The work along this line is in progress.

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